One-step Incomplete Multi-view Clustering based on Bipartite Graph Learning

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***Abstract*—Although previous graph-based multi-view clustering algorithms have made remarkable progress, most of them still face the following two limitations: 1. Many existing methods rely on k-means for the discretization of spectral embeddings, which cannot directly learn graphs with discrete cluster structures and require two steps for clustering results. 2. Practical applications may contain some missing instances, which require Incomplete Multi-View Clustering (IMVC) methods to hold them. In this paper, we propose a novel method named One-step Incomplete Multi-View Clustering based on Bipartite Graph Learning (OIMVC-BGL) which aims to solve the above problems. OIMVC-BGL first constructs bipartite graphs from all views with an anchor-based subspace learning method. Then, OIMVC-BGL fuses these graphs to obtain a consensus bipartite graph with an adaptive weight manner. Finally, OIMVC-BGL imposes a Laplacian rank constraint on the consensus bipartite graph to obtain the results directly. Experiments conducted on benchmark datasets verify the effectiveness of OIMVC-BGL.**

***Index Terms*—Unsupervised learning, Incomplete multi-view clustering, Bipartite graph, Graph fusion.**

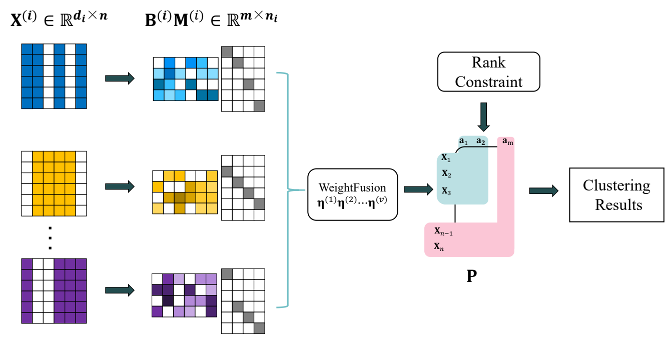

Fig. 1. The framework of OIMVC-BGL. Firstly, we construct the incomplete bipartite graph $\mathbf{B}^{(i)}\mathbf{M}^{(i)}$ on each view. Secondly, by employing adaptive weights, all bipartite graphs are fused into a consensus bipartite $\mathbf{P}$. Finally, a Laplacian rank constraint is imposed on the consensus bipartite $\mathbf{P}$ to obtain the clustering directly.

I. Introduction

Current IMVC methods can roughly be divided into the following four categories: cooperative learning-based IMVC [1], kernel-based IMVC [2], [3], NMF-based (nonnegative matrix factorization) IMVC [4], and deep learning-based IMVC [5], [6]. Among the above methods, [7] utilizes a plain collaborative training method to recover missing potential representation instances and obtain consensus representation. However, it requires separating model optimization and clustering into two independent steps, which need to implement k-means as post-processing to obtain clustering results. Moreover, the approach based on the two-step optimization cannot guarantee the global optimal kernel matrix and potential representation. [8] restructures the complete kernel from the view containing complete instances and recovers the missing element of the incomplete kernel by solving a Laplacian regularization problem. However, the technique based on Laplacian regularization can only handle a class of incomplete cases, which is not suitable for practical application. Lin *et al*. [9] propose a deep learning-based IMVC model, which can recover data and learn the representation by contrastive learning. However, high time complexity caused by the massive parameters and complex models of neural networks prevents it from being applied to large-scale tasks. This paper proposes OIMVC-BGL, which does not need any post-processing and can be well applied to large-scale data, to solve the above problems. The algorithm framework is shown in Figure 1.

We first use subspace learning to obtain the incomplete bipartite graph of each missing view. Then, we fuse them into a consensus bipartite graph and improve each base bipartite graph simultaneously. In the fusion process, view weights are learned adaptively to balance the influence of different view-specific bipartite graphs. In addition, we apply a Laplacian

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rank constraint to the consensus bipartite to obtain k-connected component numbers directly, thus obtaining clustering results. Experimental results on various benchmark datasets demonstrate the effectiveness and efficiency of OIMVC-BGL. Our appendix and code are available at https://github.com/CLDOKK/OIMVC-BGL.

II. Related work

A. Multi-view subspace learning based on anchor points

It is a challenge for multi-view subspace clustering that fuses information from different views [10]. Let $\mathbf{X}^{(v)} \in R^{d^{(v)} \times n}$ denote the data of the v-th view in multi-view data, where $d^{(v)}$ is the dimensionality of the v-th view. According to the assumptions in sparse subspace clustering (SSC) [11], all data points can be written as a linear combination of other ones. The objective function of SSC is as follows,

$$\begin{aligned} &\min_{\mathbf{Z}^{(v)}} \left\| \mathbf{X}^{(v)} - \mathbf{X}^{(v)}\mathbf{Z}^{(v)} \right\|_{\mathrm{F}}^2, \\ &s.t.\ z_{ii}^{(v)} = 0\ (\forall i), \end{aligned} \tag{1}$$

where $\mathbf{Z}^{(v)} \in \mathbb{R}^{n \times n}$ denotes the self-representation matrix. The constraint of $\mathbf{Z}^{(v)}$ guarantees each point can only be represented by other data points except itself.

In recent years, researchers usually use the $n \times m$ anchor graph (bipartite graph between original samples and anchor points) to replace the $n \times n$ similar graph to decrease the time complexity [12], [13]. An effective method [14] first constructs anchor points by performing k-means clustering on original datasets and then lets the corresponding clustering centroids be the anchor points. In comparison with traditional SSC, the computational complexity of the method that uses a bipartite graph instead of the whole graph is reduced from $O(n^2)$ to $O(nm)$.

B. One-step method on Multiview Clustering

As we know, most graph clustering methods are two-step. They fuse all base graphs to obtain a consensus graph and then explore the clustering structure from the consensus graph. Compared with the one-step method, the two-step method not only requires an additional post-process to get the final results but also affects the exploration of the true cluster structure. A pioneer work on the one-step methods is proposed in [15] named constrained Laplacian rank method (CLR). After that, [12] improves the CLR for multi-view clustering which can be formulated as

$$\begin{aligned} &\min_{\mathbf{P}, \eta^{(v)}} \left\| \sum_{v=1}^{V} \eta^{(v)} \mathbf{B}^{(v)} - \mathbf{P} \right\|_{\mathrm{F}}^2, \\ &s.t.\ \boldsymbol{\eta}^\top \mathbf{1} = 1, \boldsymbol{\eta} \geq 0, \mathbf{P}\mathbf{1} = \mathbf{1}, P_{ij} \geq 0, \\ &rank(\tilde{\mathbf{L}}_{\mathbf{S}}) = n + m - k, \end{aligned} \tag{2}$$

where $\boldsymbol{\eta} = \left[\eta^{(1)}, \eta^{(2)}, \cdots, \eta^{(v)}\right]^\top$ is the weight of each view, $\mathbf{B}^{(v)}$ is the v-th bipartite graph, $\mathbf{P} \in \mathbb{R}^{n \times m}$ is the consensus bipartite graph with Laplace rank constraint, and $\tilde{\mathbf{L}}_{\mathbf{S}}$ is a normalized Laplacian matrix of the similarity graph $\mathbf{S}$ which can be constructed as

$$\mathbf{S} = \begin{bmatrix} & \mathbf{P} \\ \mathbf{P}^\top & \end{bmatrix} \in \mathbb{R}^{(n+m)\times(n+m)}. \tag{3}$$

Lemma 1. Let $\mathbf{S}$ be a non-negative undirected graph, the multiplicative number of eigenvalue 0 of the normalized Laplacian matrix $\tilde{\mathbf{L}}_{\mathbf{S}}$ is equal to the number of connected components in the graph [15].

Motived by Lemma 1, we can directly obtain the cluster results from the similarity graph without performing k-means or other discretization procedures [15].

III. One-step Incomplete Multi-view Clustering based on Bipartite Graph Learning (OIMVC-BGL)

A. Model proposal

By performing SSC on every single view and combining them into a unified objective function, we can obtain

$$\begin{aligned} &\min_{\mathbf{Z}^{(v)}} \sum_{v=1}^{V} \left\{ \left\| \mathbf{X}^{(v)} - \mathbf{X}^{(v)}\mathbf{Z}^{(v)} \right\|_F^2 + \lambda \left\| \mathbf{Z}^{(v)} \right\|_{\mathrm{F}}^2 \right\} \\ &+ \beta \mathcal{F}(\mathbf{Z}^{(1)}, \ldots, \mathbf{Z}^{(V)}), \\ &s.t.\ \mathbf{Z}^{(v)} > \mathbf{0}, \mathbf{Z}^{(v)}\mathbf{1} = \mathbf{1}, z_{ii}^{(v)} = 0\,(\forall i), \end{aligned} \tag{4}$$

where $\left\|\mathbf{Z}^{(v)}\right\|_{\mathrm{F}}^2$ prevents the model from getting a trivial solution, $\mathcal{F}(\mathbf{Z}^{(1)}, \ldots, \mathbf{Z}^{(V)})$ strengthens the consistency of multiple self-representation matrices $\mathbf{Z}^{(v)}$, and λ, β are two trade-off hyper-parameters. Let m be the number of anchors and $\mathbf{A}^{(v)} \in \mathbb{R}^{d^{(v)} \times m}$ be the anchor matrix of the v-th view. We extend (4) from the traditional subspace learning to the subspace learning based on anchor points as follows,

$$\begin{aligned} &\min_{\mathbf{B}^{(v)}} \sum_{v=1}^{V} \left\{ \left\| \mathbf{X}^{(v)} - \mathbf{A}^{(v)}\mathbf{B}^{(v)} \right\|_{\mathrm{F}}^2 + \lambda \left\| \mathbf{B}^{(v)} \right\|_{\mathrm{F}}^2 \right\} \\ &+ \beta \mathcal{F}(\mathbf{B}^{(1)}, \ldots, \mathbf{B}^{(V)}), \\ &s.t.\ \mathbf{B}^{(v)} \geq 0, \mathbf{B}^{(v)}\mathbf{1} = \mathbf{1}, \end{aligned} \tag{5}$$

where $\mathbf{B}^{(v)} \in \mathbb{R}^{n \times m}$ is the bipartite graph between n original samples and m anchor points of the v-th view.

In practice, some accidents like sensor damage will cause absent instances of partial view and generate incomplete data. To adress this issue, we then extend the above framework for incomplete multi-view clustering (IMVC). Defining the index vector as $\mathbf{h}^{(v)} \in \mathbb{R}^{n_i}$ to represent the index of the n_i existing sample on the v-view. Then we can define the incomplete indicator matrix $\mathbf{M}^{(v)} \in \mathbb{R}^{n \times n_i}$ for the v-th view as follows,

$$M_{pq}^{(v)} = \begin{cases} 1, & if\ h_q^{(v)} = p, \forall q = 1, 2, \cdots, n_i, \\ 0, & otherwise. \end{cases} \tag{6}$$

In this way, the v-view of the incomplete sample matrix can be represented by $\mathbf{X}^{(v)}\mathbf{M}^{(v)} \in \mathbb{R}^{d^{(v)} \times n_i}$. Combining (6), we can rewrite (5) for IMVC as

$$\begin{aligned}
&\min_{\mathbf{B}^{(v)}} \sum_{v=1}^{V} \left\{ \left\| \mathbf{X}^{(v)}\mathbf{M}^{(v)} - \mathbf{A}^{(v)}\mathbf{B}^{(v)}\mathbf{M}^{(v)} \right\|_{\mathrm{F}}^2 + \lambda \left\| \mathbf{B}^{(v)} \right\|_{\mathrm{F}}^2 \right\} \\
&+ \beta \mathcal{F}(\mathbf{B}^{(1)}, \ldots, \mathbf{B}^{(V)}), \\
&s.t.\ \mathbf{B}^{(v)} \geq 0, \mathbf{B}^{(v)}\mathbf{1} = \mathbf{1}.
\end{aligned} \tag{7}$$

The quality of anchors is a serious problem in IMVC. When the missing rate is relatively large, some anchor selection strategies like uniform sampling, mean value, or k-means may result in poor clustering performance. Therefore, we use the observed samples in each base view to learn the anchor points. Specifically, we use a projection matrix $\mathbf{W}^{(v)} \in \mathbb{R}^{k \times d^{(v)}}$ to project each view into a potential space. The preliminary objective function can be written as

$$\begin{aligned}
&\min_{\mathbf{W}^{(v)}, \mathbf{A}^{(v)}, \mathbf{B}^{(v)}} \sum_{v=1}^{V} \left\{ \left\| \mathbf{W}^{(v)}\mathbf{X}^{(v)}\mathbf{M}^{(v)} - \mathbf{A}^{(v)}\mathbf{B}^{(v)}\mathbf{M}^{(v)} \right\|_{\mathrm{F}}^2 \right. \\
&\left. + \lambda \left\| \mathbf{B}^{(v)} \right\|_{\mathrm{F}}^2 \right\} + \beta \mathcal{F}(\mathbf{B}^{(1)}, \ldots, \mathbf{B}^{(V)}), \\
&s.t.\ \mathbf{B}^{(v)} \geq 0, \mathbf{B}^{(v)}\mathbf{1} = \mathbf{1}, \mathbf{W}^{(v)}\mathbf{W}^{(v)\top} = \mathbf{I}_k, \\
&\mathbf{A}^{(v)}\mathbf{A}^{(v)\top} = \mathbf{I}_k.
\end{aligned} \tag{8}$$

In the fusion process, Equation (2) is used to fuse bipartite graphs of multiple views into a consensus bipartite graph. According to *lemma 1*, the multiplicity of eigenvalue 0 is equal to the number of connected components in the graph. Thus, a similar matrix with k-connected components is obtained, and the samples in the same connected branch can be treated as a cluster. Since (3), the connection of $\mathbf{P}$ is the same as $\mathbf{S}$. So that the clustering results can be obtained directly from the bipartite graph $\mathbf{P}$.

We apply the unified bipartite graph learning and bipartite graph fusion to IMVC. Equation (8) is combined with (2), where (2) can be considered as a regularization term. Then the Laplacian rank constraint bridges the gap between bipartite graph learning and discrete cluster structure learning. At the same time, a projection matrix with dimension k is added for feature screening. Formally, the objective function of unified discrete bipartite graph learning can be expressed as follows,

$$\begin{aligned}
&\min_{\mathbf{W}^{(v)}, \mathbf{A}^{(v)}, \mathbf{B}^{(v)}, \mathbf{P}, \boldsymbol{\eta}} \sum_{v=1}^{V} \left\{ \left\| \mathbf{W}^{(v)}\mathbf{X}^{(v)}\mathbf{M}^{(v)} - \mathbf{A}^{(v)}\mathbf{B}^{(v)}\mathbf{M}^{(v)} \right\|_{\mathrm{F}}^2 \right. \\
&\left. + \lambda \left\| \mathbf{B}^{(v)} \right\|_{\mathrm{F}}^2 \right\} + \beta \left\| \sum_{v=1}^{V} \eta^{(v)}\mathbf{B}^{(v)} - \mathbf{P} \right\|_{\mathrm{F}}^2, \\
&s.t.\ \mathbf{A}^{(v)}\mathbf{A}^{(v)\top} = \mathbf{I}_k, \mathbf{B}^{(v)} \geq 0, \mathbf{B}^{(v)}\mathbf{1} = \mathbf{1}, \\
&\boldsymbol{\eta}^\top \mathbf{1} = 1, \boldsymbol{\eta} \geq 0, \mathbf{W}^{(v)}\mathbf{W}^{(v)\top} = \mathbf{I}_k, \\
&\mathbf{P}\mathbf{1} = \mathbf{1}, P_{ij} \geq 0, rank(\tilde{\mathbf{L}}_{\mathbf{S}}) = n + m - k.
\end{aligned} \tag{9}$$

The details of the optimization and algorithm process are included in Section 1 of the appendix.

B. Time complexity analysis

The computational complexity of OIMVC-BGL consists of five optimization steps. For the v-view, the main calculation steps include: Update $\mathbf{W}^{(v)}$, which computational complexity is $O(d^{(v)}(nm{+}km{+}k^2))$; Update $\mathbf{A}^{(v)}$, which computational complexity is $O(nmk + nd^{(v)}k + d^{(v)2}k)$; Update $\mathbf{P}$, which computational complexity is $O(nmkt + nm^2t + m^3t)$, and t is number of iterations ; Update $\mathbf{B}^{(v)}$, which computational complexity is $O(nm^2d^{(v)})$; Update $\boldsymbol{\eta}$, which computational complexity is $O(V^2nm)$. Each variable update is about the linear time complexity of n. So the overall computational complexity of OMIVC-BLG is linear.

IV. Experiment

A. Datasets and Baselines

Four multi-view datasets are used in the experiment: Prokaryotic [16], Caltech101-7 [17], NUSWIDE [18], and YoutubeFace. Prokaryotic includes text data, gene lineage, and protein composition information of 551 prokaryotes in four categories. Caltech101-7 is a subset of the image dataset Caltech101. NUSWIDE is an object recognition dataset with 30000 instances. YoutubeFace is a video dataset collected from YouTube with 101499 instances.

To compare the results of the algorithm, we employ seven algorithms as a baseline to compare with OIMVC-BGL on datasets. BSV [19] fills all missing views with the sample mean of corresponding views, then uses the best views as clustering results. Concat [20] concat all views of each sample into a feature vector. FLSD [21] learns potential view-specific representations and seek shared representations of clusters based on semantically consistent constraints. UEAF [22] learns a uniform cluster representation while recovering missing views. DAIMC [23] introduces a view algorithm-specific weight matrix to solve the missing view problem and align the base matrix. IMVC-CBG [24] is a scalable anchor graph framework is proposed to solve the IMVC problem for the first time. FIMAVC-VIA [25] is an IMVC method with fast processing for large-scale partial data. To make better use of specific information, it learns individual anchors on each view.

B. Experimental Setting

For all of the compared algorithms, we implement them by downloading their public Matlab code from the website. All parameters of compare algorithms conform to the description of the corresponding literature. For OIMVC-BGL, we use grid search in the range of $\left[10^{-3}, 10^{-2}, \cdots 10^{2}, 10^{3}\right]$ to obtain the best hyperparameters λ and β. Similarly, the number of anchor m is chosen by grid search in the range of $[k, 2k, 4k, 6k, 8k]$. All experiments are performed on a desktop with Ubuntu 22.04.2LTS, 64-bit operating system, 13th Gen Intel(R) Core(TM) i7-13700KF x24 CPU 64GIB, MATLAB R2021b.

TABLE I
CLUSTERING RESULTS IN THE METRIC OF ACCUARCY(ACC) OF DIFFERENT METHODS WITH DIFFERENT MISSING RATES (0.5, 0.3, AND 0.0).THE BEST RESULT IS REPRESENTED BY BOLDFACE, THE SECOND-BEST THE RESULT IS REPRESENTED BY AN UNDERLINE, AND N/A MEANS OUT OF THE CPU MEMORY

Datasets	Prokaryotic			Caltech101-7			NUSWIDE			YouTube		
Missing rate	0.5	0.3	0.0	0.5	0.3	0.0	0.5	0.3	0.0	0.5	0.3	0.0
BSV	63.36	57.93	60.47	50.09	61.53	66.39	8.61	8.18	8.76	N/A	N/A	N/A
Concat	40.37	43.64	46.93	42.63	40.69	45.22	12.77	13.51	15.60	N/A	N/A	N/A
FLSD	46.85	44.93	44.10	38.17	39.35	42.96	N/A	N/A	N/A	N/A	N/A	N/A
UEAF	57.95	57.79	43.74	36.40	46.80	40.77	N/A	N/A	N/A	N/A	N/A	N/A
DAIMC	46.35	55.47	56.98	45.42	50.08	48.24	14.23	14.57	14.78	N/A	N/A	N/A
IMVC-CBG	55.17	57.89	59.53	70.37	73.39	66.46	14.39	14.99	14.60	15.93	22.11	23.37
FIMAVC-VIA	62.73	63.60	63.40	48.53	65.63	50.35	12.90	13.27	13.55	15.19	17.21	18.87
Ours	**68.78**	**70.05**	**73.50**	**77.68**	**78.36**	**83.79**	**18.54**	**18.94**	**19.78**	**26.64**	**26.63**	**26.64**

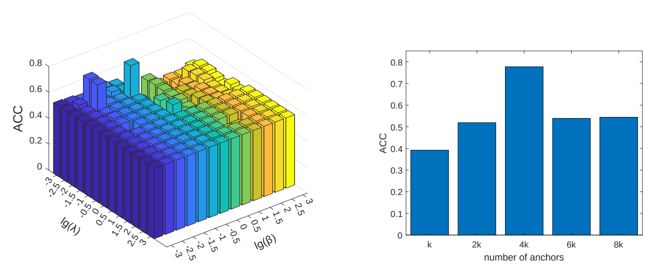
TABLE II
THE RUNNING TIME OF DIFFERENT METHODS FOR PROCESSING DATASETS (UNIT: S)

Datasets	Prokaryotic	Caltech101-7	NUSWIDE	YouTube
BSV	0.07	0.77	16.87	N/A
Concat	0.22	5.67	202.06	N/A
FLSD	0.65	20.87	N/A	N/A
UEAF	0.65	7.68	N/A	N/A
DAIMC	6.73	99.06	911.95	N/A
IMVC-CBG	0.46	1.25	28.43	175.79
FIMAVC-VIA	0.12	0.59	23.60	91.22
Ours	0.17	4.16	62.16	110.32

C. Experimental Results and Analysis

As shown in Table 1, OIMVC-BGL achieves the best performance compared with all comparison algorithms. OIMVC-BGL can effectively deal with large-scale datasets such as YouTube and also have great performance on small-scale datasets such as prokaryotic. As an algorithm based on bipartite graph learning, the comparison with the IMVC-CBG and the FIMVC-VIA proves the effectiveness of the one-step method.

The time spent is a crucial performance measure, particularly for the method based on bipartite graph learning. Table 2 shows the running time of multiple algorithms. Although OIMVC-BGL is not the fastest one among various algorithms, it can obtain clustering results in a short period. Existing IMVC methods, such as IMVC-CBG, rely on k-means as a post-process to obtain the final clustering results. Instability is one of the disadvantages of k-means. So many researchers employ at least 20 times k-means and then average the results to obtain relatively stable results, which spend more time in the experiment. OIMVC-BGL transforms the multiple k-means into a simple iteration of the parameter, which can obtain stable results quickly.



(a) (b)

Fig. 2. The analysis of parameters on the Caltech7 dataset. (a) The parameters analysis of λ and β. (b) The parameters analysis of the number of anchors.

D. Parameters Analysis

In OIMVC-BGL, hyper-parameters λ, β and the number of anchors required to be set. To explore the influence of parameter selection, we separate fixing one of them to conduct comparative experiments. Figure 2 shows the analysis of parameters with a 0.5 missing rate on Caltech101-7 dataset. From the analysis, our method is relatively stable in different parameters and appropriate parameter values facilitate obtaining better clustering results.

V. CONCLUSION

In this paper, a novel IMVC method termed OIMVC-BGL is proposed. OIMVC-BGL uses subspace learning based on the anchor to obtain view-specific bipartite graphs then merge them into a consistent bipartite graph with adaptive view weights. By combining with Laplacian low-rank constraints, discrete cluster structures can be obtained directly from fusion graphs. Unlike existing IMVC methods, OIMVC-BGL can deal with large-scale IMVC tasks effectively without postprocessing. The comparative experiments on real datasets demonstrate the superiority of OIMVC-BGL.

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